## Supplementary Materials for "Compressive single-pixel imaging with low-order nonlinearity neural networks"

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## A. The mathematical relationship between inputs and labels in three regression tasks

In this paper, whether a regression task is linear or nonlinear relies on the mathematical relationship between the input and label. The following will introduce the methods for generating the labels of three regression tasks.

For imaging task, the object x from the MNIST or Fashion MNIST datasets serves as the network input and label simultaneously. But for the linear-edge extraction task, the label is the corresponding linear Sobel edge:

$$Lab_{\text{linear}} = S_h \cdot x + S_v \cdot x \tag{S1}$$

where  $\cdot$  represents the convolution operation, x,  $S_h$  and  $S_v$  are the object, the Sobel horizontal and vertical edge filters respectively. The two edge filters can be expressed as:

$$S_h = \begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix}, \quad S_v = \begin{bmatrix} -1 & -2 & 1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{bmatrix}$$
 (S2)

Similarly, the labels for nonlinear edge extraction tasks can be written as:

$$Lab_{\text{nonlinear}} = \sqrt{(S_h \cdot x)^2 + (S_v \cdot x)^2}$$
 (S3)

## B. The experimental results of nonlinear tasks based on $3_{\rm rd}\text{-}{\rm order}$ PNN

To further verify the fitting capability of the PNN, two nonlinear tasks, nonlinear-edge extraction and handwritten dataset classification, were conducted experimentally based on the  $3_{\rm rd}$ -order PNN. The results were presented in Fig. S1, with the average PSNR/SSIM of  $22.72 {\rm dB}/0.87$  and the accuracy of 95%.

In Fig. S1a, the images, reconstructed from complete Hadamard basis patterns and undergoing nonlinear-edge operators, serves as the ground truth of nonlinear-edge extraction tasks. The quantitative assessment metrics, PSNR and SSIM, are marked in the lower right corner of restored images. Besides that, the confusion matrix of handwritten dataset classification is presented in Fig. S1b with an accuracy of 95%. Compared with the  $2_{\rm nd}$ -order experimental results of two nonlinear tasks, presented in the main text, it can be observed that the  $2_{\rm nd}$ -order PNN is already capable of effectively extracting the desired features, which implies that the corresponding ONN will have a simple structure.

## C. Optical hardware implementation of $2_{\rm nd}$ -order PNN

The Fig. S2a presents the schematic diagram of optical hardware implementation of  $2_{\rm nd}$ -order PNN. The 1D incoherent light field z, serving as the bucket signals, is split into two beams, with each beam propagating through an optical matrix-vector multiplier(OMVM). After modulated by multipliers, the Hadamard product between the two modulated light fields( $y_1$  and  $y_2$ ) is performed by the OASLM. Then, the combination of  $y_1$  and the output light of OASLM undergoes a final linear transformation to produce the output y. A classical optical prototype of matrix-vector multiplier is shown in Fig. S2b, which can be implemented with a 4f-type system composed of two cylindrical lens(CL). Furthermore, Fig. S2c presents the diagram of the OASLM.

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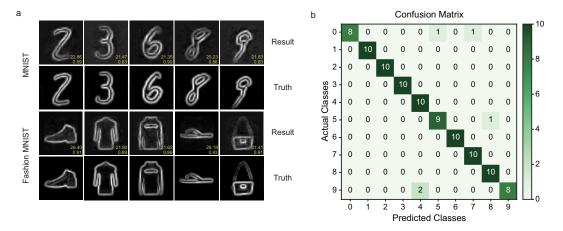


FIG. S1. Experimental results of two nonlinear tasks based on  $3_{rd}$ -order PNN. a The results of nonlinear-edge extraction task, with the PSNR and SSIM showed in the lower right corner of reconstructed images. b The confusion matrix of handwritten dataset classification with an accuracy of 95%.

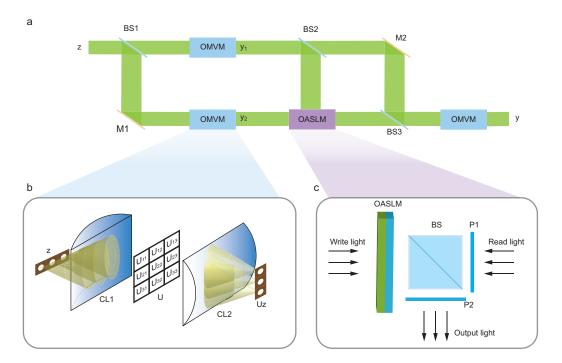


FIG. S2. Schematic of ONN of  $2_{nd}$ -order PNN. a The 1D incoherent light field z, serving as the bucket signals, is split into two beams, with each beam propagating through an optical matrix-vector multiplier. After modulated by multipliers, the Hadamard product between the two modulated light fields( $y_1$  and  $y_2$ ) is performed by the optically addressed spatial light modulators. Then, the combination of  $y_1$  and the output light of OASLM undergoes a final linear transformation to produce the output y. b A classical optical prototype of matrix-vector multiplier, which can be implemented with a 4f-type system composed of two cylindrical lens. c The diagram of the OASLM. BS: beam splitter, M: mirror, OMVM: optical matrix-vector multiplier, OASLM: optically addressed spatial light modulators CL: cylindrical lens, P: polarizer.